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## **Spherical Cavity-Expansion Forcing Function in PRONTO 3D for Application to Penetration Problems**

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# **Spherical Cavity-Expansion Forcing Function in PRONTO 3D for Application to Penetration Problems**

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## **Abstract**

In certain penetration events the primary mode of deformation of the target can be approximated by known analytical expressions. In the context of an analysis code, this approximation eliminates the need for modeling the target as well as the need for a contact algorithm. This technique substantially reduces execution time. In this spirit, a forcing function which is derived from a spherical-cavity expansion analysis has been implemented in PRONTO 3D. This implementation is capable of computing the structural and component responses of a projectile due to three dimensional penetration events. Sample problems demonstrate good agreement with experimental and analytical results.

## **Acknowledgments**

The authors gratefully acknowledge numerous helpful discussions with Dr. M. Forrestal and Dr. D. Longcope at Sandia National Laboratories, Albuquerque, NM and Dr. M. Adley at US Army Corps of Engineers Waterways Experiment Station, Vicksburg, MS.

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# Nomenclature

$a$	cavity radius
$b_1$	first bounding coordinate
$b_2$	second bounding coordinate
$c_1$	constant term nodal pressure coefficient
$c_2$	linear term nodal pressure coefficient
$c_3$	quadratic term nodal pressure coefficient
$f'_c$	unconfined compressive strength of concrete
$m$	mass of the projectile
$\bar{n}$	unit outward normal vector
$p_I$	pressure at node $I$
$\bar{V}_I$	velocity vector at node $I$
$\vartheta$	target particle velocity at the nose-target interface
$A$	constant term dimensionless fitting coefficient
$B$	linear term dimensionless fitting coefficient
$C$	quadratic term dimensionless fitting coefficient
$E$	Young's modulus
$I$	integer variable with range 1 to 4 corresponding to element side nodes
$N$	geometric parameter
$P$	depth of penetration
$S$	empirical dimensionless constant
$V_s$	striking velocity of the projectile
$Y$	quasi-static yield strength
$\nu$	Poisson's ratio
$\rho_o$	density of the undeformed target material
$\rho_p$	density of the undeformed projectile material
$\sigma_r$	radial stress at the cavity surface
$\bar{\sigma}_r$	average radial stress at the cavity surface
$\psi$	caliber-radius-head (CRH)

# **Spherical Cavity Expansion Forcing Function in PRONTO 3D for Application to Penetration**

## **Introduction**

Computational modeling of penetration and perforation remains to be an active field of research. A literature search on this topic reveals numerous recent publications that propose new methods or improve existing ones. There has been considerable effort and progress in the development of various codes (based on different representations of the conservation laws for a continuum: Lagrangian, Eulerian, Arbitrary Lagrangian-Eulerian, etc.) that serve as powerful and versatile computational tools which are routinely used to solve complex problems. However, at present, the time required to complete a single penetration run is still excessive and prohibits any prospect for streamlining the penetration analysis for use in an overall design tool that would permit numerous simulations.

In cases where the deformation mode of the target can be captured (to first order) by a known analytical model and where the primary interest is in the structural response of the projectile, the penetration analysis can then be reduced to modeling the projectile by itself. The target is replaced by a known forcing function that approximates its resistance. This eliminates the need to model the target and furthermore eliminates the need for a contact algorithm, which leads to substantial savings in execution time.

Forrestal et al. (1988) recognized that the resistance produced by an aluminum target could be approximated by a dynamic cavity-expansion analysis. They developed closed-form expressions for the depth of penetration of rigid projectiles with different nose shapes and demonstrated good agreement with experimental results. The same concept is applied here, but in the context of a three dimensional finite element code. A forcing function based on the dynamic expansion of a spherical cavity is implemented in the Sandia developed explicit dynamic finite element code PRONTO 3D (Taylor and Flanagan, 1987) and can be utilized by analysts who have access to the PRONTO 3D computer code. This implementation is capable of handling a full three dimensional penetration event which

includes: oblique impact, nonzero angle of attack, nonlinear deformations of the projectile, response of components internal to the projectile, etc. The accuracy of this method depends on how well the forcing function approximates the actual situation; however, in many cases the spherical cavity expansion (which is derived on the basis of an unbounded medium) does provide a good approximation for events where the free surface effects are minimal. Thus, this implementation is most accurate for cases of deep penetration.

This method has previously been applied with some success using cavity expansion forcing functions with beam elements in the general purpose finite element code ABAQUS implicit (Hibbitt, Karlsson, and Sorensen, Inc., 1989) by Longcope (1991) and Longcope (1996), and using empirical forcing functions with shell elements in ABAQUS implicit by Adley and Moxley (1996), with shell elements in ABAQUS explicit by Duffey and Macek (1997), and also with tetrahedron, brick, and shell elements in EPIC 97 (Johnson et al., 1997).

## Analytical Spherical Cavity Expansion

Analytical methods for penetration mechanics began with the work of Bishop, Hill, and Mott (1945). They developed equations for the quasi-static expansions of cylindrical and spherical cavities and used these equations to estimate forces on conical nose punches pushed slowly into metal targets. Goodier (1965) developed a model to predict the penetration depth of rigid spheres launched into metal targets. His penetration model included target inertial effects, so he approximated the target response by results from the dynamic, spherically symmetric, cavity-expansion equations for an incompressible target material derived by Hill (1948) and discussed by Hill (1950) and Hopkins (1960). In more recent work (Forrestal et al., 1988; Forrestal et al., 1991; Forrestal et al., 1995; Forrestal and Tzou, 1996; and Warren and Forrestal, 1997) it has been shown that the radial stress at the cavity surface obtained from spherical cavity-expansion models can be accurately represented by a function of the form

$$\frac{\sigma_r(a)}{Y} = A + B \left( \sqrt{\frac{\rho_o}{Y}} \vartheta \right) + C \left( \sqrt{\frac{\rho_o}{Y}} \vartheta \right)^2, \quad (1)$$



where  $v$  is the target particle velocity at the cavity-target interface,  $a$  is the cavity radius,  $Y$  is the quasi-static yield strength of the target material,  $\rho_o$  is the density of the undeformed target material, and  $A$ ,  $B$ , and  $C$  are dimensionless fitting coefficients. The expression given in (1) is also consistent with the semi-empirical model developed by Forrestal et al. (1994) for penetration into concrete targets.

## Implementation of the Spherical Cavity Expansion Forcing Function in PRONTO 3D

The spherical cavity-expansion forcing function is implemented in PRONTO 3D as a normal traction (or pressure) boundary condition that acts on a prescribed surface. This new option has been added to the PRONTO 3D command language and is invoked with the command line:

CAVity EXPAntion, side set id AXIS=direction BOUNDs= $b_1, b_2$  COEF= $c_1, c_2, c_3$

where uppercase letters represent the minimum abbreviation of each word and

side set id	must match a side set on the GENESIS file
direction	X, Y, or Z (default=Z)
$b_1, b_2$	bounding coordinates along the direction specified in AXIS
$c_1, c_2, c_3$	constant nodal pressure coefficients.

The command line shown above can be repeated, as needed, in order to represent layers with different parameters.

Four nodal pressures are calculated in PRONTO 3D for each element side (i.e. a side of a hexagonal continuum element or mid-surface of a structural shell element) included in the side set as shown in Fig. 1. These nodal pressures are obtained from

$$p_I = c_1 + c_2(\vec{V}_I \cdot \vec{n}) + c_3(\vec{V}_I \cdot \vec{n})^2 \quad (I = 1, 4), \quad (2)$$

where the dot represents a scalar product,  $\vec{V}_i$  is the nodal velocity vector,  $\vec{n}$  is the outward unit vector normal to the diagonals of the side, and the constant nodal pressure coefficients are related to the dimensionless fitting coefficients in (1) as  $c_1 = AY$ ,  $c_2 = B(\rho_o Y)^{1/2}$ , and  $c_3 = C\rho_o$ . The values of  $p_i$  are updated during each time increment using the current values of  $\vec{V}_i$  and  $\vec{n}$ . If the scalar product  $(\vec{V}_i \bullet \vec{n})$  at a node is zero, negative, or if the node lies outside the bounds set by  $b_1$  and  $b_2$  then the pressure is set to zero for that node.

A set of consistent global forces arising from these pressures over an element side are calculated as discussed by Taylor and Flanagan (1987). These forces are accumulated as each element side in the side set is considered.

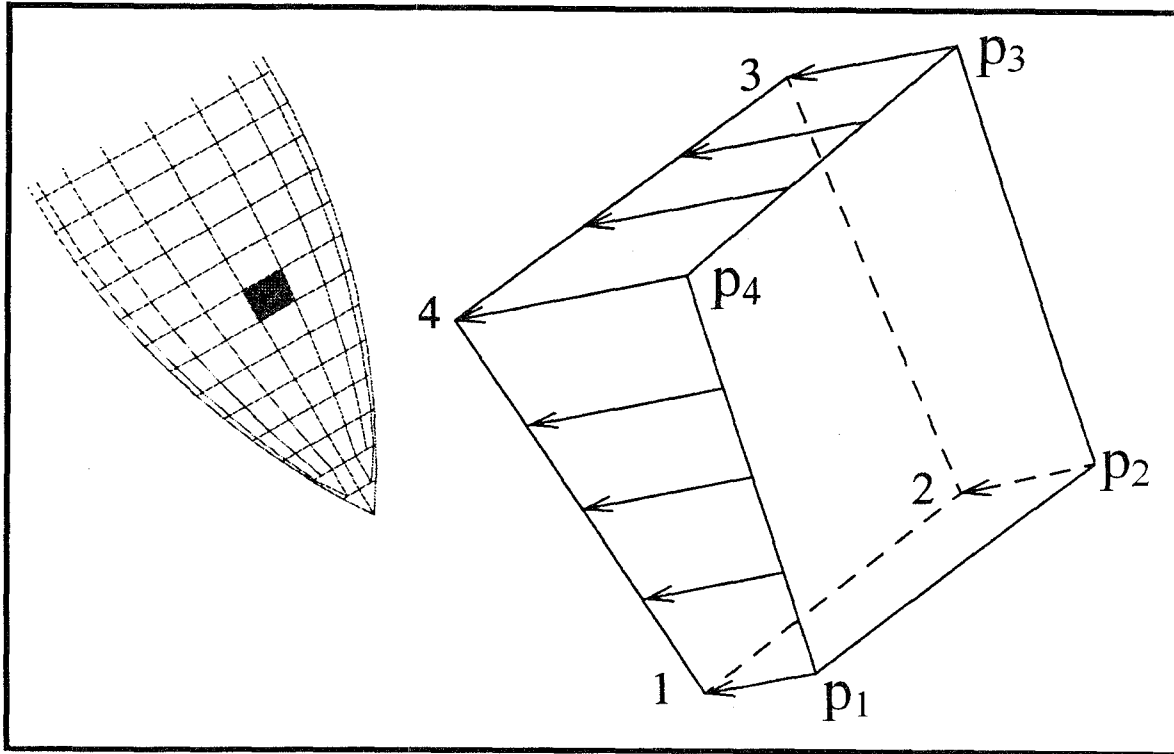


Figure 1. Definition of a pressure boundary condition that acts on an element side.

## Numerical Examples

In this section we present example problems of penetration into both aluminum and concrete targets which demonstrate the significant features of the cavity expansion forcing function option in PRONTO 3D. In all of the example problems we only provide the input instructions and not the geometric definitions of the mesh.

### Penetration into Aluminum Targets

In these examples we consider the penetration of 6061-T651 aluminum targets by solid spherical-nose, C-300 maraging steel rods launched at striking velocities between 350 and 1200 m/s. These rods have density  $\rho_p = 8000 \text{ kg/m}^3$ , shank length  $L=71.12 \text{ mm}$ , nose radius  $a=3.55 \text{ mm}$ , and nominal mass of 0.0235 kg. The target is modeled as a compressible, strain hardening and strain-rate sensitive material for which the undeformed density, quasi-static yield strength, and dimensionless fitting coefficients required for use with (1) are given by Warren and Forrestal (1997) as  $\rho_o = 2710 \text{ kg/m}^3$ ,  $Y=276 \text{ Mpa}$ ,  $A=5.0394$ ,  $B=0.9830$ , and  $C=0.9402$  respectively. The finite element mesh used in the following examples is shown in Fig. 2 and is comprised of 3172 nodes and 2784 elements.

We first compare results obtained by PRONTO 3D with the experimentally verified analytical model given by Warren and Forrestal (1997) in order to validate the cavity expansion forcing function. In the analytical model the projectile is assumed to be rigid; therefore, we use the rigid material model in PRONTO 3D. An example input file for a striking velocity of 960 m/s is shown in Fig. 3. The cavity-expansion command line includes a side set id of 100 which represents the surfaces of the nose and shank (excluding the rear of the projectile). A bounding coordinate,  $b_1=0$  is used to define the free surface and  $b_2=-10 \text{ m}$  to reflect an unbounded medium. Figure 4 shows the depth of penetration at several striking velocities which are in good agreement with the analytical results. A termination time of 350  $\mu\text{s}$  was used for each of the striking velocities, requiring approximately 22 cpu seconds on a CRAY J-90.

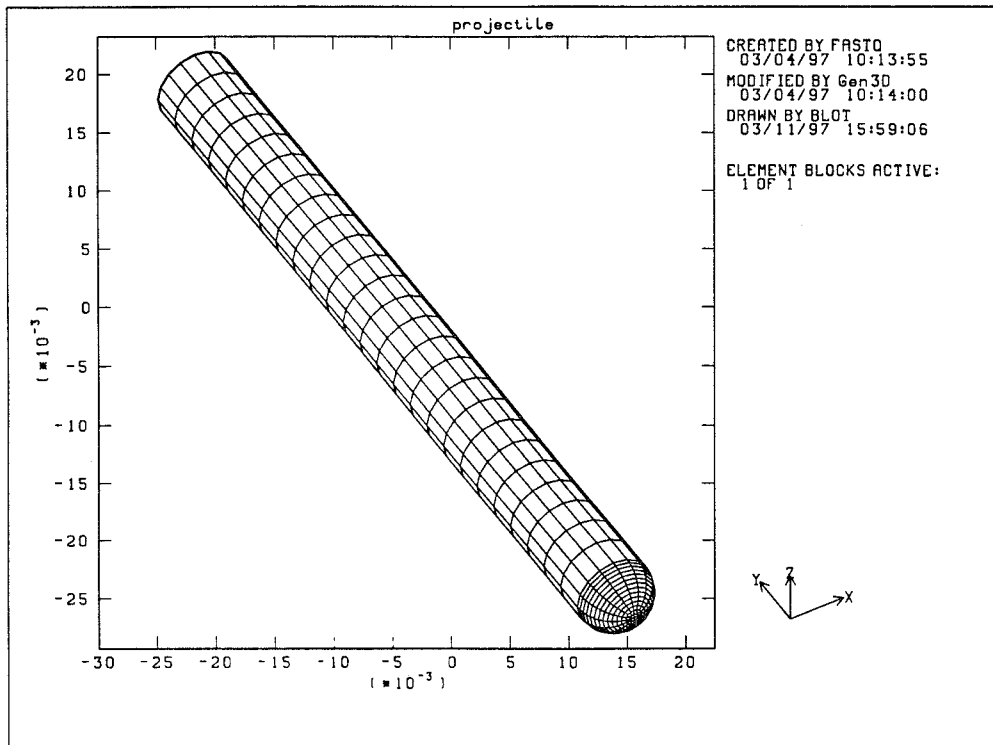


Figure 2. Finite element mesh of the spherical-nose rod.

```

title
penetration into aluminum
material,1,rigid,8000.
contact modulus = 1
end
rigid time step = 3.5e-7
time step scale = 1
bulk viscosity 0,0
initial velocity material,1 0,-960,0
cavity expansion, 100 axis=Y bounds=0,-10 coef=1.39087e9,8.50144e5,2.54794e3
termination time=0.00035
plot time=1.4e-5
history time =7.0e-6
plot nodal= displ,velocity,mass
plot element=eqps,vonmises
plot history coord=0,0,0 vari=velo comp=y name=a
plot history coord=0,0,0 vari=disp comp=y name=a
exit

```

Figure 3. PRONTO 3D input file for the rigid projectile problem.

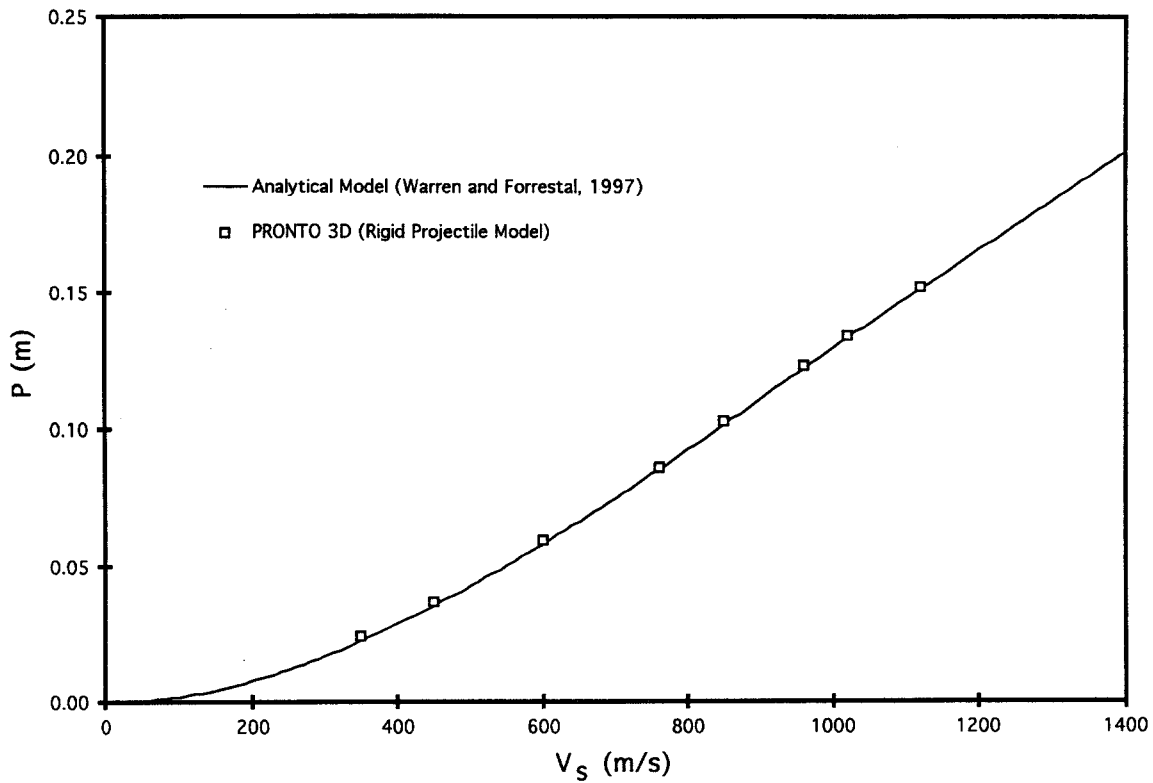


Figure 4. Depth of penetration versus striking velocity for a rigid projectile.

Next, we assume the C-300 maraging steel projectiles to behave as an elastic, linear hardening material and use the elastic-plastic material model in PRONTO 3D. The Young's modulus, yield strength and Poisson's ratio for maraging steel are given by The International Nickel Company, Inc. (1964) as  $E = 189$  GPa,  $Y = 2.067$  GPa, and  $\nu = 0.3$  respectively. Experimental results obtained by Chait (1972) from compression tests of C-300 maraging steel at room temperature and a strain rate of  $3 \times 10^{-4} \text{ s}^{-1}$  give a strain hardening modulus of  $E' = 177.7$  MPa. An example input file using these parameters is shown in Fig. 5 for a striking velocity of 960 m/s. Depth of penetration results are compared in Fig. 6 with the rigid projectile analytical solution of Warren and Forrestal (1997), and also with experimental data obtained by Forrestal et al. (1988) for C-300 maraging steel and by Forrestal et al. (1991) for T-200 maraging steel which is slightly softer than the C-300 maraging steel. Good agreement is observed between the analytical solution and the PRONTO 3D solution for striking velocities below 700 m/s; however, for higher velocities the projectile deformation reduces the depth of penetration (it should

```

title
penetration into aluminum
material,1,elastic plastic ,8000.
youngs modulus = 189.0e9
poissons ratio = 0.3
yield stress = 2.067e9
hardening modulus = 1.777e8
beta = 1.0
end
time step scale = 1
bulk viscosity 0,0
initial velocity material,1 0,-960,0
cavity expansion, 100 axis=Y bounds=0,-10 coef=1.39087e9,8.50144e5,2.54794e3
termination time=0.00035
plot time=1.4e-5
history time =7.0e-6
plot nodal= displ,velocity,mass
plot element=eqps,vonmises
plot history coord=0,0,0 vari=velo comp=y name=a
plot history coord=0,0,0 vari=disp comp=y name=a
exit

```

Figure 5. PRONTO 3D input file for an elastic-plastic projectile.

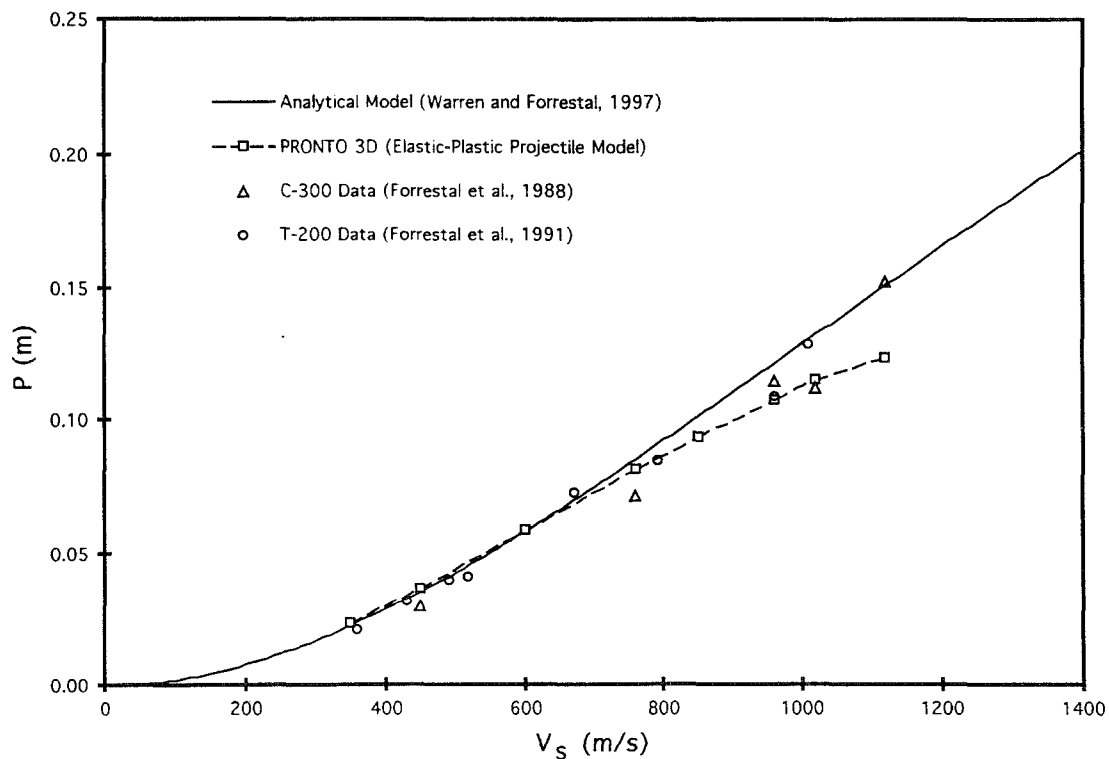


Figure 6. Depth of penetration versus striking velocity for an elastic-plastic projectile.

be noted that the depth of penetration would be greater if strain-rate effects had been included in the constitutive model for the penetrator). A sequence of deformed penetrator configurations is shown in Fig. 7 for a striking velocity of  $V_s=1120$  m/s. It is observed that the penetrator bulges slightly early in the penetration event requiring it to open a larger

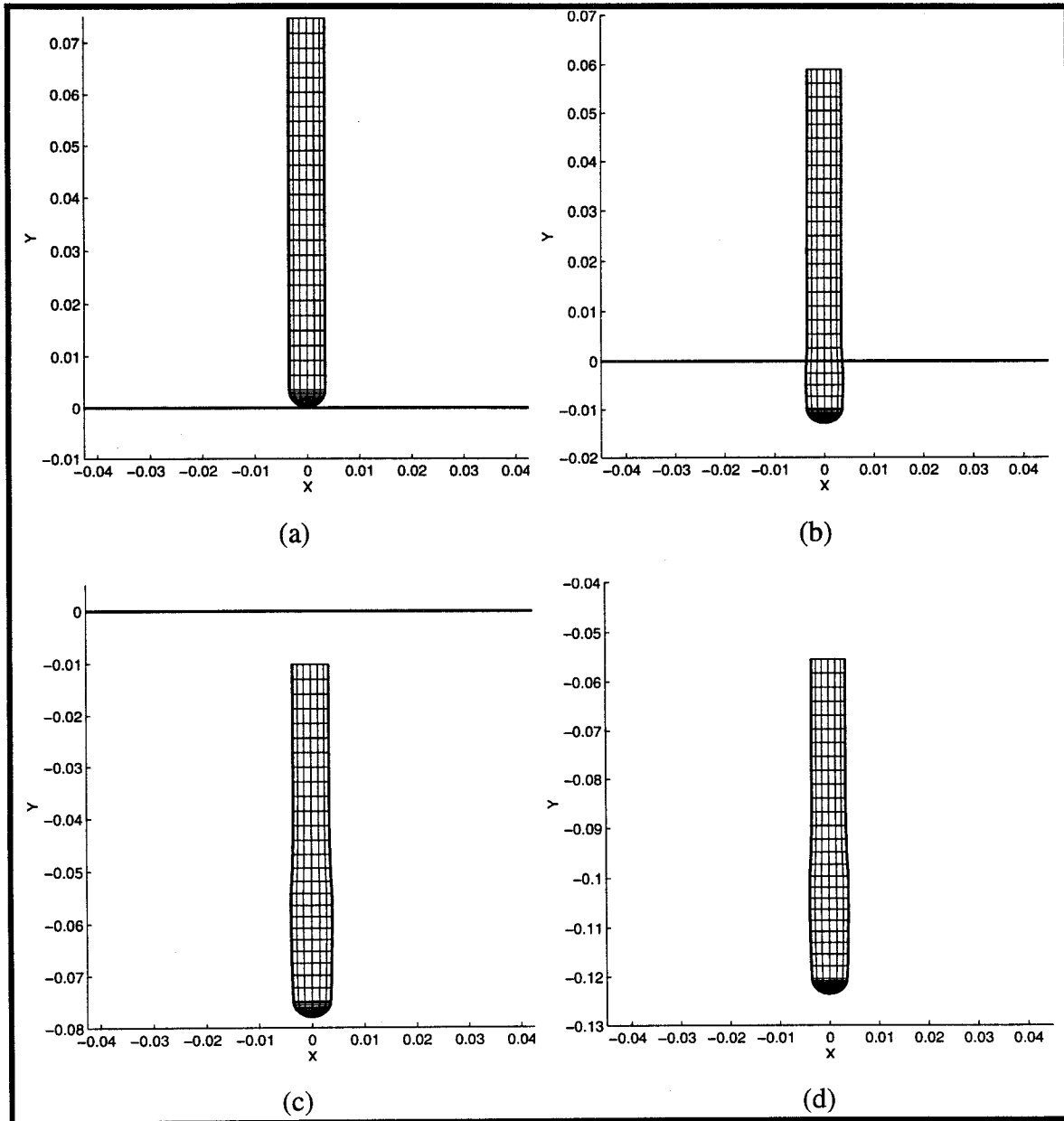


Figure 7. Projectile deformation for a striking velocity  $V_s=1120$  m/s at : (a) 0.0 s, (b) 14  $\mu$ s, (c) 98  $\mu$ s, and (d) 280  $\mu$ s.

cavity which reduces the depth of penetration. A termination time of 350  $\mu$ s was used for each of the striking velocities, requiring approximately 1482 cpu seconds on a CRAY J-90.

As another example we consider the oblique impact of a C-300 maraging steel projectile with a 6061-T651 aluminum target. The input file for a 30 degree oblique impact with a zero angle of attack and a striking velocity of 960 m/s is shown in Fig. 8. A sequence of deformed penetrator configurations for this striking velocity and angle of obliquity is shown in Fig. 9. It is observed that the projectile initially bulges slightly and starts to bend due to the non-symmetric loading. The projectile continues to bend and rotate throughout the penetration event until it finally comes to rest. As in the previous example, a termination time of 350  $\mu$ s was used requiring approximately 1482 cpu seconds on a CRAY J-90. While we have good agreement between data and predictions for normal impacts, at this time we have no oblique impact data for aluminum targets.

```
title
penetration into aluminum
material,1,elastic plastic ,8000.
youngs modulus = 189.0e9
poissons ratio = 0.3
yield stress = 2.067e9
hardening modulus = 1.777e8
beta = 1.0
end
time step scale = 1
bulk viscosity 0,0
initial velocity material,1 480.0,-831.4,0
cavity expansion, 100 axis=Y bounds=0,-10 coef=1.39087e9,8.50144e5,2.54794e3
termination time=0.00035
plot time=1.4e-5
history time =7.0e-6
plot nodal= displ,velocity,mass
plot element=eqps,vonmises
plot history coord=0,0,0 vari=velo comp=y name=a
plot history coord=0,0,0 vari=disp comp=y name=a
exit
```

Figure 8. PRONTO 3D input file for an oblique impact of 30 degrees.



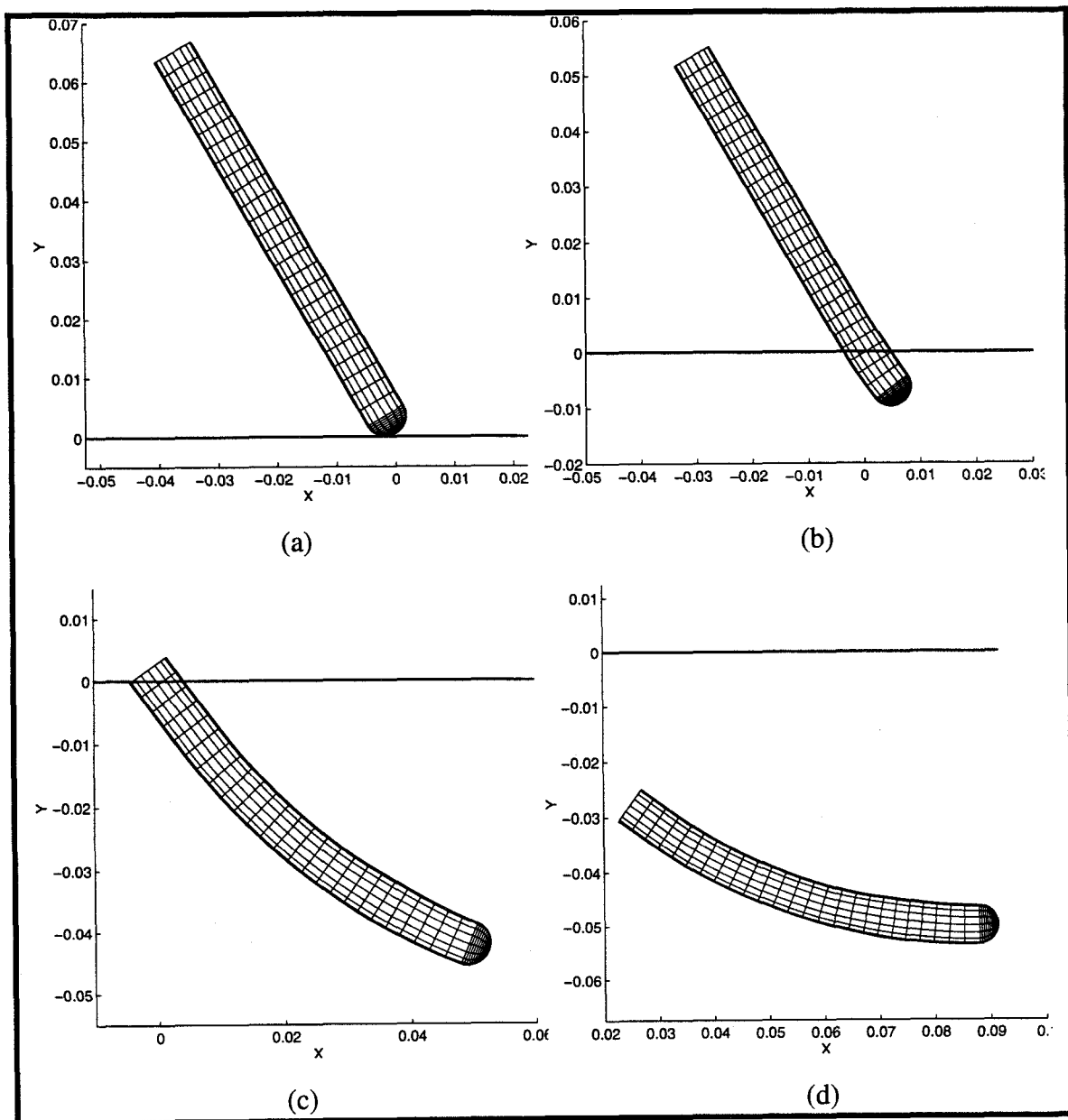


Figure 9. Projectile deformation for an oblique impact of 30 degrees and a striking velocity  $V_s=960$  m/s at : (a) 0.0 s, (b) 14  $\mu$ s, (c) 98  $\mu$ s, and (d) 252  $\mu$ s.

## Penetration into Concrete Targets

In this example we consider the penetration of 58.4 Mpa (8.5 ksi) concrete targets by solid 3.0 caliber-radius-head (CRH) ogive-nose, 4340 R<sub>c</sub> 45 steel rods launched at striking velocities between 400 and 1200 m/s. These rods have density  $\rho_p = 7830 \text{ kg/m}^3$ , shank length  $L=169.5 \text{ mm}$ , shank diameter  $2a=20.3 \text{ mm}$ , nose length  $l=33.7 \text{ mm}$ , and nominal mass of 0.478 kg. The coefficients in (1) are obtained using the semi-empirical method developed by Forrestal et al. (1994) for penetration into concrete. The finite element mesh used in this example is shown in Fig. 10 and is comprised of 3197 nodes and 2816 elements.

As discussed by Forrestal et al. (1994) there are two regions in the penetration process of concrete. The first region is a conical cratering region with a depth of approximately two projectile diameters. The second region is the tunneling region which starts at the end of the cratering region and proceeds to the final depth of penetration. For the cratering region we average the expression for the stress (Forrestal et al., 1994) acting on the penetrator over the range  $0 \leq y \leq 4a$  which gives

$$\bar{\sigma}_r = \frac{1}{4a} \int_0^{4a} \sigma_r dy = \frac{m}{8\pi a^3} \left[ V_s^2 - \frac{mV_s^2 - 4\pi a^3 S f'_c}{m + 4\pi a^3 N \rho_o} \right], \quad (3)$$

where  $m$  is the mass of the projectile,  $V_s$  is the striking velocity,  $f'_c$  is the unconfined compressive strength of the target,  $\rho_o$  is the density of the undeformed target material,  $S$  is an empirical dimensionless constant, and  $N$  is a geometric parameter defined by

$$N = \frac{8\psi - 1}{24\psi^2}, \quad (4)$$

with  $\psi$  being the CRH number. Thus, the value for the constant  $A$  in the cratering region is directly obtained from the relation  $\bar{\sigma}_r = A f'_c$  (i.e.  $B=C=0$ ), where we take  $Y = f'_c$  and for

the given target material  $f'_c = 58.4$  MPa,  $S=9.037$ , and  $\rho_o = 2320$  kg/m<sup>3</sup>. In the tunneling region  $A=S$ ,  $B=0.0$ , and  $C=1.0$  (Forrestal et al., 1994).

We assume the 4340 R<sub>c</sub> 45 steel projectiles to behave as an elastic-plastic power-law hardening material and use the isotropic elastic-plastic power-law hardening material model in PRONTO 3D. The density, Young's modulus, yield strength and Poisson's ratio for 4340 steel projectiles are given by Luk and Piekutowski (1991) as  $\rho = 7810$  kg/m<sup>3</sup>,  $E = 206.8$  GPa,  $Y = 1.207$  GPa, and  $\nu = 0.32$  respectively. Curve fitting the data in Luk and Piekutowski (1991) gives a hardening constant of 382 Mpa and hardening exponent of 0.266 which are required for use in the material model. An example input file for a striking velocity of 1162 m/s is shown in Fig. 11. Depth of penetration results are compared in Fig. 12 with the semi-empirical analytical solution of Forrestal et al. (1994), and with experimental data obtained by Frew et al. (1997). Good agreement is observed with both the analytical solution and experimental data. . A termination time of 3000  $\mu$ s was used for each of the striking velocities, requiring approximately 3785 cpu seconds on a CRAY J-90.

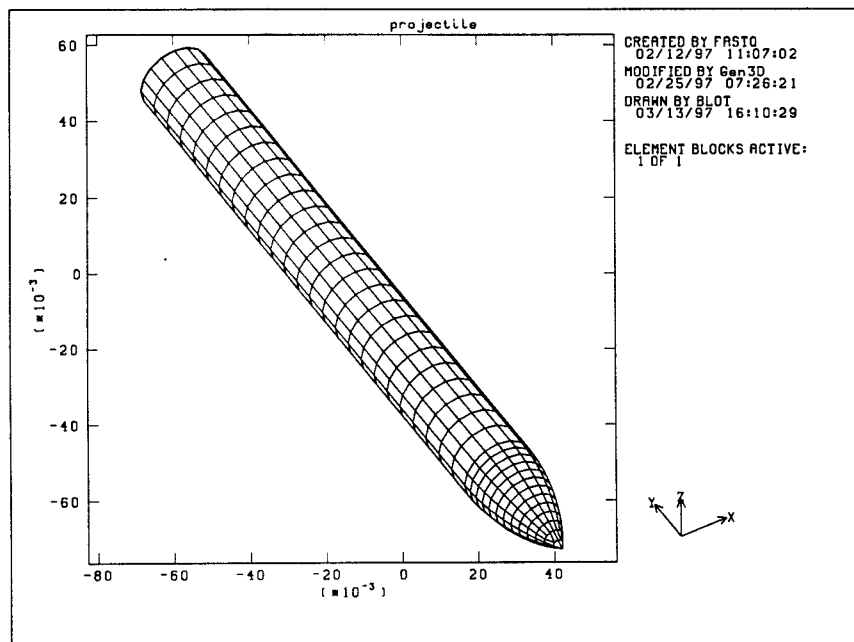


Figure 10. Finite element mesh of the ogive-nose rod.

```

title
penetration into concrete
material,1,ep power hard, 7810.
youngs modulus = 206.8e9
poissons ratio = 0.32
yield stress = 1.207e9
hardening constant = 3.8247155277e8
hardening exponent = 0.2962824
luders strain = 0.0
end
bulk viscosity 0,0
initial velocity material,1 0,-1162.0,0
cavity expansion, 100 axis=Y bounds=0,-0.0406 coef=4.27757e8,0,0.0
cavity expansion, 100 axis=Y bounds=-0.0406,-10 coef=5.277e8,0,2.320e3
termination time=3.0e-3
plot time=1.2e-4
plot element = pressure,eqps,vonmises
plot nodal = displacement,velocity,mass
plot history coord=0,0,0 vari=velo comp=y name=a
plot history coord=0,0,0 vari=disp comp=y name=a
history time=6e-5
exit

```

Figure 11. PRONTO 3D input file for the elastic-plastic power-law hardening projectile.

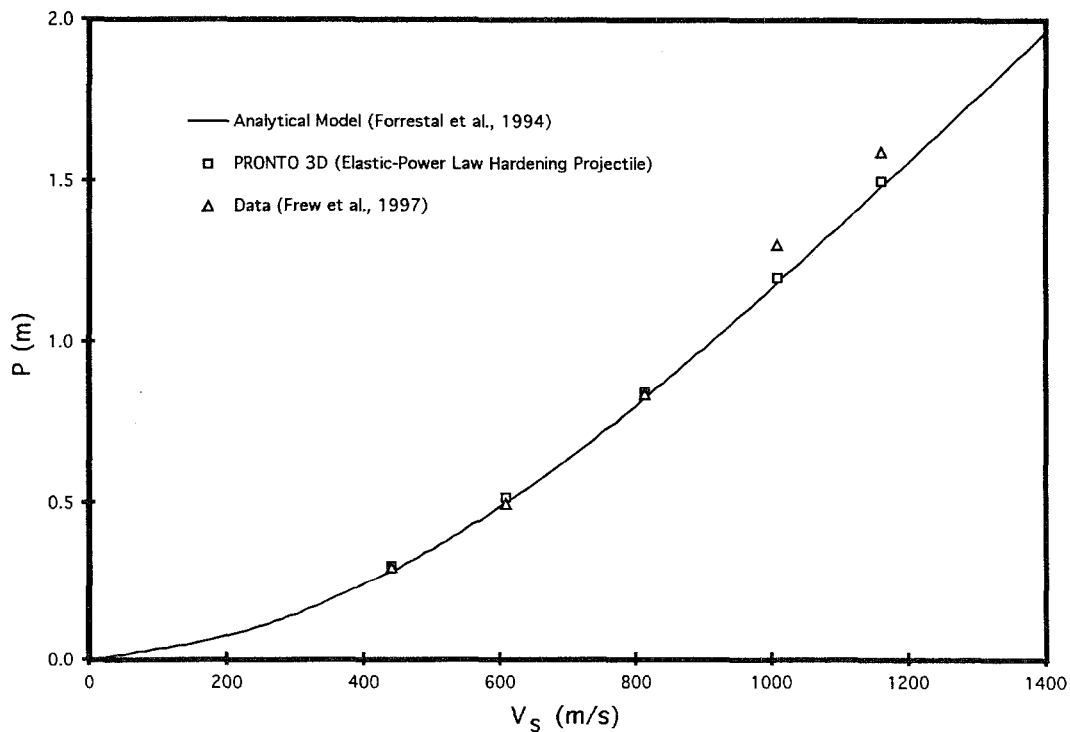


Figure 12. Depth of penetration versus striking velocity for an elastic plastic projectile.

## Conclusion

A spherical cavity-expansion forcing function has been implemented into PRONTO 3D. This new capability is intended for analyzing three dimensional penetration events with multi-layered targets. Simulations of normal penetration into aluminum and concrete targets showed good agreement with analytical and experimental results. For the class of problems within the realm of assumptions considered here, this technique is efficient and robust and can be included in a broader software tool utilized for design purposes. Since PRONTO 3D is a Lagrangian code with explicit time integration, good accuracy is expected in computing the structural response of projectiles with nonlinear constitutive models.

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